Geometric and Analytic Number Theory — *programme*

Monday Nov 19, 2018

8:30 – 9:00. Welcome and Registration

9:00 – 9:50. Ian Petrow, *The Weyl bound for Dirichlet L-functions of cube-free conductor*

Coffee

10:45 – 11:15. Peter Humphries, *The random wave conjecture for dihedral Maass forms*

11:30 – 12:20. Farrell Brumley, *Concentration properties of theta lifts*

Lunch

15:00 – 15:30. Alisa Sedunova, *Jordan totient quotients*

Coffee

16:15 – 17:05. Lilian Matthiesen, *On rational points in fibrations and multiplicative functions*

Tuesday Nov 20, 2018

9:00 – 9:50. Étienne Fouvry, *The level of distribution of unbalanced convolution*

Coffee

10:45 – 11:15. Lasse Grimmelt, *Vinogradov's Theorem with Fouvry-Iwaniec Primes*

11:30 – 12:20. Damaris Schindler, *On prime values of binary quadratic forms with a thin variable*

Lunch

15:00 – 15:30. Rainer Dietmann, *Enumerative Galois theory for quartics*

Coffee

16:15 – 17:05. Simon Myerson, *New directions for forms in many variables*

Wednesday Nov 21, 2018

9:00 – 9:50. Paul Nelson, *tba*

Coffee

10:45 – 11:15. Ramon Moreira Nunes, *The twelfth moment of Dirichlet L-functions to smooth moduli*

11:30 – 12:20. Abhyshek Saha, *Sup-norms of Maass forms on compact arithmetic surfaces in the level aspect*

Lunch

15:00 – 15:30. Efthymios Sofos, *Mixed moments of additive functions evaluated at thin subsets of* $\mathbb{Z}^n$

Coffee

16:15 – 17:05. Gergely Harcos, *On the global sup-norm of* $GL(n)$ *cusp forms*

Thursday Nov 22, 2018

9:00 – 9:50. Jürg Kramer, *An arithmetic Riemann-Roch formula on modular curves via heat kernel degeneration*

Coffee

10:45 – 11:15. Giuliano Gagliardi, *Existence of equivariant models of spherical homogeneous spaces*

11:30 – 12:20. Anna von Pippich, *The special value $Z'(1)$ of the Selberg zeta function for the modular group*

Lunch

15:00 – 15:30. Lola Thompson, *Counting quaternion algebras*

Coffee

16:15 – 17:15. Zeev Rudnick, *Angles of Gaussian primes*

Friday Nov 23, 2018

9:00 – 9:50. Dan Loughran, *Number fields with prescribed norms*

Coffee

10:45 – 11:35. Christopher Frei, *Average bounds for l-torsion in class groups*

12:00 – 12:45. Trevor Wooley, *l^r-decoupling and efficient congruencing*

Abstracts

Brumley: *Concentration properties of theta lifts*
I will present some results, obtained jointly with Simon Marshall, on the concentration properties of automorphic forms obtained through the theta correspondence. The first result examines pointwise concentration on quotients of real hyperbolic grassmannians. As these spaces are of higher rank, the lower bound takes on an interesting shape, which we hope sheds lights on a refinement of Sarnak's purity conjecture. A companion result on rank one hyperbolic spaces produces eigenfunctions correlating, in a quantifiable sense, with Gaussian beams.

Dietmann: *Enumerative Galois theory for quartics.*
We consider monic quartic polynomials with integer coefficients and growing box height at most H. In this setting, we exactly determine the order of magnitude (from above and below) of such polynomials whose Galois group is D_4. Moreover, we show that C_4 and V_4 polynomials are less frequent that D_4 ones, and that D_4, C_4, V_4 and A_4 polynomials are together less frequent than reducible quartics. In particular, irreducible non-$S4$ quartics are less numerous than reducible ones, for the first time solving a case (namely degree four) of a conjecture by van der Waerden from 1936.

Frei: *Average bounds for l-torsion in class groups.*
We discuss average upper bounds for the size of the l-torsion of class groups of some families of number fields, for arbitrary positive integers l. In particular, we focus on the families of degree-d-fields for d between 2 and 5. This is joint work with Martin Widmer.

Fouvry: *The level of distribution of unbalanced convolution*
We give new examples of convolutions of two arithmetic sequences with an exponent of distribution greater than 1/2. The particular feature of these results is the fact that the support of one of these sequences is much smaller than the other one. We will present some applications to classical questions of equidistribution. This work is in collaboration with M. Radziwill.

Gagliardi: *Existence of equivariant models of spherical homogeneous spaces*
Let G be a connected reductive group. Over any algebraically closed field of characteristic 0, a complete combinatorial description of spherical G-varieties is known. In this talk, we obtain necessary and sufficient conditions for the existence of models of spherical homogeneous spaces over arbitrary fields of characteristic 0 (joint work with Mikhail Borovoi).

Harcos: *On the global sup-norm of $GL(n)$ cusp forms*
The sup-norm problem, a close relative of the subconvexity problem, has been studied widely for the group GL(2). There are fewer results for GL(n), and it was only recently that an explicit upper bound was derived for the global sup-norm of unramified Maass cusp forms there. I will discuss this result, which is joint work with Valentin Blomer and Péter Maga.

Humphries: *The random wave conjecture for dihedral Maass forms*
I discuss two problems on arithmetic quantum chaos associated to dihedral Maass forms: Planck scale mass equidistribution and the asymptotic for the fourth moment. This is joint work with Rizwanur Khan.

Kramer: *An arithmetic Riemann-Roch formula on modular curves via heat kernel degeneration.*
In our talk, we present a variant of an arithmetic Riemann-Roch theorem for the line bundle of modular forms of even weight on modular curves equipped with the Petersson metric. Since this metric is known to have logarithmic singularities at the cusps, such a statement cannot be directly derived from known results in the smooth setting. However, we can take the known arithmetic Riemann-Roch formula in the smooth setting as a starting point, and then approximate the singular hyperbolic metric under consideration by smooth metrics. A careful study of the degeneration behavior of the hyperbolic heat kernel allows us in the end to regularize the starting relation and to derive a formula involving special values of the Selberg zeta function.

Loughran: *Number fields with prescribed norms.*
We consider the problem of counting abelian number fields for which a given rational number is a norm. This is joint work with Christopher Frei and Rachel Newton.

Matthiesen: *On rational points in fibrations and multiplicative functions.*
In this talk I will describe progress on Serres problem on the number of varieties in a family that have a rational point. Our methods include tools from additive combinatorics as well as results on multiplicative functions. This is joint work with Dan Loughran.

Myerson: *New directions for forms in many variables*
A classic result of Birch estimates the number of solutions to a system of equations $f = 0$ in integers of size B or less. Here f consists of R forms of degree d with integral coefficients which is in a suitable sense nonsingular. The theorem applies if the number of variables is large: of size at least $C(d)R^2$ for some constant $C(d)$. Recently a number of sophisticated variants of Birch's method have been developed. These count (among others) solutions in semiprime variables, almost prime variables or smooth variables as well as linear spaces of solutions. In previous work we reduced the number of variables in Birch's original result to $C'(d)R$. We discuss a refinement of these techniques, and possible future applications.

Nunes: *The twelfth moment of Dirichlet L-functions to smooth moduli.*
We show how to obtain an analogue for Dirichlet L-functions with smooth moduli of Heath-Brown's estimate of the twelfth moment of the Riemann-zeta function. Our result relies on bounds for some exponential sums that we study by appealing to the formalism of algebraic trace functions due to Katz and Fouvry-Kowalski-Michel.

Petrow: *The Weyl bound for Dirichlet L-functions of cube-free conductor.*
In the 1920s Weyl proved the first non-trivial estimate for the Riemann zeta function on the critical line: $\zeta(1/2 + it) \ll (1 + |t|)^{1/6+\epsilon}$. The analogous bound for a Dirichlet L-function $L(1/2, \chi)$ of conductor q as q tends to infinity is still unknown in full generality. In a breakthrough around 2000, Conrey and Iwaniec proved the analogue of the Weyl bound for $L(1/2, \chi)$ when χ is assumed to be quadratic of conductor q. Building on the work of Conrey and Iwaniec, we show (joint work with Matt Young) that the Weyl bound for $L(1/2, \chi)$ holds whenever its conductor is cube-free. The method also applies to some GL_2 L-functions which are not twist-minimal.

von Pippich: *The special value $Z'(1)$ of the Selberg zeta function for the modular group*
In this talk we report on an explicit formula for the special value at $s = 1$ of the derivative of the Selberg zeta function for the modular group $\Gamma = \mathrm{PSL}_2(\mathbb{Z})$. The formula is a consequence of a generalization of the arithmetic Riemann-Roch theorem of Deligne and Gillet-Soulé to the case of the trivial sheaf of $\Gamma \setminus \mathbb{H}$, equipped with the hyperbolic metric. This is joint work with Gerard Freixas.

Rudnick: *Angles of Gaussian primes.*
Fermat showed that every prime $p \equiv 1 \bmod 4$ is a sum of two squares: $p = a^2 + b^2$, and hence such a prime gives rise to an angle whose tangent is the ratio b/a. Do these angles exhibit order or randomness? I will discuss the statistics of these angles and present a conjecture, motivated by a random matrix model and by function field considerations. (joint work with Ezra Waxman)

Schindler: *On prime values of binary quadratic forms with a thin variable*
A result of Fouvry and Iwaniec states that there are infinitely many primes of the form x^2+y^2 where y is a prime number. In this talk we will see a generalization of this theorem to the situation of an arbitrary primitive positive definite binary quadratic form. This is joint work with Peter Cho-Ho Lam and Stanley Xiao.

Sedunova: *Jordan totient quotients.*
The Jordan totient function is a natural generalisation of the Euler totient function. We study their average behaviour and its connections to the logarithmic derivatives of cyclotomic polynomials.

Sofos: *Mixed moments of additive functions evaluated at thin subsets of $\mathbb{Z}^n$.*
Assume that we are given n additive functions $g_1, \dots, g_n$ defined on the positive integers. We study the mixed moments

$$\sum_{\mathbf{x} \in A} \prod_{i=1}^{n} g_i(|x_i|)^{k_i},$$

where k_i are arbitrary positive integers and A is a subset of $\mathbb{Z}^n$. As an application we obtain multivariate analogues of the Erdős–Kac theorem for subsets of $\mathbb{Z}^n$ that are the zero locus of various polynomial systems. The talk is based on joint work with Daniel El-Baz.

Thompson: *Counting quaternion algebras*
We discuss how classical techniques from analytic number theory can be used to count quaternion algebras over number fields subject to various constraints. Because of the correspondence between maximal subfields of quaternion algebras and geodesics on arithmetic hyperbolic manifolds, these counts have interesting applications to the field of spectral geometry. This talk is based on a joint paper with B. Linowitz, D. B. McReynolds, and P. Pollack.

Wooley: *l^r-decoupling and efficient congruencing*
We describe an interpretation of efficient congruencing as l^2-decoupling, and generalise to l^r-decoupling with r a positive real number between 1 and 2. Our results generalise those focused on investigations surrounding Vinogradovs mean value theorem as pursued in recent work of the speaker and work of Bourgain, Demeter and Guth. This is joint work with Kevin Hughes. If time permits, we will discuss conclusions of similar flavour focused on mean value estimates associated with Waring's problem.